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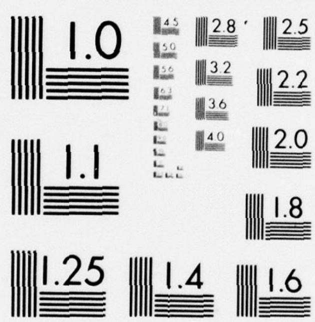
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CONTROL OF DYNAMICAL SYSTEMS

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Sept. 1, 1978 - March 1, 1979

Lefschetz Center for Dynamical Systems
Division of Applied Mathematics
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Providence, R. I. 02912

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March 17, 1979

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H. T. Banks

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Six papers were published during the period of this report. Efforts were continued on approximation of delay systems with applications to optimal control and parameter identification. Studies are being made on the effects of delays on difference and differential-difference equations. Research has been centered on the stability properties of infinite-dimensional systems. Work has been completed on illustrating the effect of nonlinearities of controllability and stability and the tradeoffs required in order to meet control objectives and constraints on the state variables.

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I. Identification and control of systems with delays

(H. T. Banks)

Banks is continuing his efforts on approximation of delay systems with applications to optimal control and parameter identification. In a manuscript [1] that has just been accepted for publication, Banks and Kappel developed the foundations for general spline approximation schemes for linear functional differential equations. Banks, in [2] and in joint efforts with Burns and Cliff [3], has tested these approximations (with simple numerical examples) when applied to control and parameter identification problems with linear delay systems. Banks is also currently conducting numerical tests with the spline approximation ideas for nonlinear control systems.

For the parameter identification problems, sound theoretical results for some of the approximation schemes under investigation still remain to be developed,especially when the system delays themselves must be identified. We have begun work on some questions arising in this connection and are making good progress.

Banks and a graduate student, Rosen, have begun efforts on development of fully discretized methods (as opposed to those of [1], [2], [3] which involve approximation of the infinite dimensional delay systems by a finite number of ordinary differential equations). We have just begun our efforts on the theoretical aspects of approximating linear delay systems in this manner.

Some initial progress has been made, but completion of this theory, numerical experiments, and application to control and identification problems are efforts still to be pursued in the near future.

II. Qualitative theory of functional differential and difference equations (J. K. Hale)

Hale is continuing his study of the effects of delays on difference and differential-difference equations. For the difference equation

$$x(t) - \sum_{j=1}^{N} A_k x(t-\gamma_k \cdot r) = 0$$

$$\gamma_k \cdot r = \sum_{j=1}^{M} \gamma_{k_j} r_j, \quad \gamma_{k_j} \geq 0, \quad \text{integers},$$

Avellar and Hale [4] have given a rather complete characterization of the behavior of the set

$$\overline{Z}(r,A) = \{\mathrm{Re}\lambda : \det[I - \sum_{k=1}^{N} A_k e^{-\lambda\gamma_k \cdot r}] = 0\}$$

as a function of the delays r and the coefficients A. Also, effective computational criterion have been given. Specific emphasis is given the stable case where $\overline{Z}(r,A) \subseteq (-\infty,0)$. The results have already been applied by Hale and de Oliveira [5]

to the Hopf bifurcation in difference equations and by de Oliveira [6] to neutral equations.

Hale, Infante and Tsen have essentially completed a preliminary study of the implication of these results to determine the region in the coefficient space for which all solutions of the delay equation

$$\frac{dx}{dt} = B_0 x(t) + \sum_{k=1}^{N} B_k x(t-\gamma_k \cdot r)$$

approach zero as $t \to \infty$, independent of the delays. (See also the comments below.)

III. Stability of infinite-dimensional systems

(E. F. Infante)

Infante and collaborators have continued to pursue research centered on the stability properties of infinite-dimensional systems, with particular emphasis on certain problems arising in certain applications.

In [7] Infante, in cooperation with J. A. Walker of Northwestern University, has completed a study of the stability properties of a model of incompressible simple fluids with fading memory. This model consists of a functional-partial differential equation, which is viewed as a nonlinear abstract evolution equation in an appropriate Hilbert space. Liapunov stability methods are used

in this investigation.

In cooperation with J. K. Hale and a student, F.P. Tsen, Infante [8] has essentially completed an investigation on the effects of changes in the delays on the stability properties of linear neutral difference-differential equations of the form

$$\frac{d}{dt}[x(t) + \sum_{k=1}^{n} B_k x(t-\tau_k)] = A_0 x(t) + \sum_{k=1}^{n} A_k x(t-\tau_k).$$

The effect of changes of the values of the $\{\tau_k\}_{k=1}^n$ on the asymptotic behavior of the solutions of such equations is most important; indeed, it is not necessarily the case that the asymptotic behavior varies continuously with changes in the $\{\tau_k\}_{k=1}^n$. Moreover, the computation of the effect of such changes is most awkward and complicated since it involves the computation of zeros of exponential polynomials. The purpose of the investigation reported in [8] was to characterize in an appropriate and convenient manner those matrices $\{B_k\}_{k=1}^n$, A_0 and $\{A_k\}_{k=1}^n$ such that the solutions of the above equation do not change their asymptotic behavior irrespective of the values of the delays $\{\tau_k\}_{k=1}^n$. Such a characterization seems important in the design and control of systems with delays; if the model of the system falls within the characterized class, it will display considerable robustness; if it does not, considerable care must be exercised in analyzing modeling errors.

Infante, in cooperation with a student, L. Carvalho, has recently completed an investigation [9] of the stability properties of pure linear difference equations of the form

$$x_t(0) = \sum_{k=1}^{n} A_k x_t(-\tau_k),$$

where the $\{\tau_k\}_{k=1}^{n}$ are not necessarily rationally related, and therefore the solution of such equations cannot be viewed in a finite-dimensional space. By viewing the solutions of such problems in a convenient Hilbert space, sufficient conditions for the existence of a particular type of Liapunov functionals, which prove stability and asymptotic stability for such equations irrespective of the value of the delays, were found. Moreover, a characterization of the $\{A_k\}_{k=1}^{n}$ for which such functionals exist is also presented. Such results seem of importance not only for stability considerations but also for optimization purposes, since most often the functionals one is required to minimize are precisely of the form of the Liapunov functionals considered.

IV. Control and stability of discrete systems

(J. P. LaSalle)

LaSalle has spent considerable time investigating a particularly simple and instructive model of a discrete control

system. The point of this exercise is to illustrate the affect of nonlinearities on controllability and stability and the trade-offs required in order to meet control objectives and constraints on the state variables. A paper on this example is currently being prepared.

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[1] H.T. Banks and F. Kappel, Spline approximations for functional differential equations, to appear in J. Diff. Eqns.

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[4] C.E. Avellar and J.K. Hale, On the zeros of exponential polynomials, to appear in JMAA.

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[7] E.F. Infante and J.A. Walker, A stability investigation for an incompressible simple fluid with fading memory, to appear in Archives of Rational Mechanics and Analysis.

[8] J.K. Hale, E.F. Infante and F.P. Tsen, Stability in delay equations for all delays, to be submitted.

[9] L. Carvalho and E.F. Infante, On Liapunov functionals for linear difference equations, to be submitted.

PUBLICATIONS SUPPORTED BY THE AIR FORCE

SEPTEMBER 1978 - PRESENT

[1] H.T. Banks (with J.A. Burns and E.M. Cliff)

Spline-based approximation methods for control and identification of hereditary systems, to appear, Proc. Intl. Symp.onSystems Optimization & Anal., IRIA, France, Dec.11-13, 1978.

[2] H.T. Banks (with Franz Kappel)

Spline approximations for functional differential equations, Journal of Differential Equations (to appear).

[3] J.K. Hale (with J.C.F. deOliveira)

Hopf bifurcation for functional equations, Journal of Mathematical Analysis and Applications (to appear).

[4] J.K. Hale (with C.E. Avellar)

On the zeros of exponential polynomials, Journal of Mathematical Analysis and Applications (to appear).

[5] J.K. Hale (with Junji Kato)

Phase space for retarded equations with infinite delay. Funkcialaj Ekvacioj, vol. 21, no. 1, December 1978.

[6] P. Massatt

Some properties of condensing maps, Annali di Matematica Pura e Applicada (to appear).

Spline-Based Approximation Methods

for Control and Identification of Hereditary Systems

H. T. Banks, J. A. Burns and E. M. Cliff

Abstract

Convergence results for schemes based on first-order spline approximations are presented for optimal control and parameter identification problems involving linear delay-differential equations. Examples with numerical results which demonstrate the attractiveness of the proposed methods are given.

SPLINE APPROXIMATIONS
FOR
FUNCTIONAL DIFFERENTIAL EQUATIONS

H. T. Banks and F. Kappel

Abstract: We develop an approximation framework for linear hereditary systems which includes as special cases approximation schemes employing splines of arbitrary order. Numerical results for first and third order spline based methods are presented and compared with results obtained using a previously developed scheme based on averaging ideas.

HOPF BIFURCATION FOR FUNCTIONAL EQUATIONS

J.K. HALE AND J.C.F. deOLIVEIRA

Abstract: The purpose of this paper is to study the existence of a smooth Hopf bifurcation for functional equations. The bifurcation parameters may include the delays. The results will be described for a special case of the equations considered.

ON THE ZEROS OF EXPONENTIAL POLYNOMIALS

Cerino E. Avellar and Jack K. Hale

ABSTRACT: Suppose $r = (r_1, \ldots, r_M)$, $r_j \geq 0$, $\gamma_{kj} \geq 0$ integers, $k = 1,2,\ldots,N$, $j = 1,2,\ldots,M$, $\gamma_k \cdot r = \sum_j \gamma_{kj} r_j$. The purpose of this paper is to study the behavior of the zeros of the function

$$h(\lambda, a, r) = 1 + \sum_{j=1}^{N} a_j e^{-\lambda \gamma_j \cdot r}$$

where each a_j is a real number. More specifically if $\overline{Z}(a,r) = \text{closure}\{\text{Re}\ \lambda : h(\lambda,a,r) = 0\}$, we study the dependence of $\overline{Z}(a,r)$ on a, r. This set is continuous in a but generally not in r. However, it is continuous in r if the components of r are rationally independent. Specific criterion to determine when $0 \notin \overline{Z}(a,r)$ are given. Several examples illustrate the complicated nature of $\overline{Z}(a,r)$.

The results have immediate implication to the theory of stability for difference equations

$$x(t) - \sum_{k=1}^{M} A_k x(t-r_k) = 0$$

where x is an n-vector, since the characteristic equation has the form given by $h(\lambda,a,r)$. The results give information about the preservation of stability with respect to variations in the delays.

The results also are fundamental for a discussion of the dependence of solutions of neutral differential difference equations on the delays. These implications will appear elsewhere.

PHASE SPACE FOR RETARDED EQUATIONS WITH INFINITE DELAY

Jack K. Hale and Junji Kato

Abstract: It is the purpose of this paper to examine initial data from a general Banach space. We develop a theory of existence, uniqueness, continuous dependence, and continuation by requiring that the space B only satisfies some general qualitative properties. Also, we impose conditions of B which will at least indicate the feasibility of a qualitative theory as general as the one presently available for retarded equations with finite delay in the space of continuous functions. In particular, this will imply that bounded orbits should be precompact and that the essential spectrum of the solution operator for a linear autonomous equation should be inside the unit circle for $t > 0$. Also, we impose conditions which imply the definitions of asymptotic stability in R^n and B are equivalent and that the ω-limit set of a precompact orbit for an autonomous equation should be compact, connected and invariant.

SOME PROPERTIES OF CONDENSING MAPS

Paul Massatt

Abstract: In 1939, Kuratowskii introduced a measure of noncompactness of bounded sets in a metric space, called the Kuratowskii measure of noncompactness, or α-measure. This along with the associated notion of an α-contraction, has proved useful in several areas of differential equations. The principle results of this paper will be to generalize several of the results of Cooperman to more general measures of noncompactness as well as for certain set mappings. The proofs are more elementary than the ones in Cooperman. However, the basic lemma used by Cooperman which depended so much on properties of α-measures is not generalized. In fact, we give an example showing that it will not generally be valid for arbitrary measures of noncompactness.